

AP Calculus AB Summer Assignment

This assignment must be turned in the first day of class, August 12, 2024. The only exception is those who are new to the district in August. If this is the case, it will be due on Monday August 26, 2024. It will count as a quiz grade and will be graded on effort and completion.

1. Please check out Calculus Single Variable by Larson and Edwards from the Media Center.
2. Please have the attached Pre-Calculus review problems with all work shown for the non-calculator section.
3. Memorize the Unit Circle in Radians (all coordinates and angles in radians, Quiz most likely first week of school you will have 2 minutes to fill it in)
4. You must have your own graphing calculator (if you do not have one, you may check one out from the media center)
5. You must have the domain, range, end behavior and asymptotes memorized for all functions including your transcendentals. You will be expected graph these functions. There will be a test on this the second week of school.

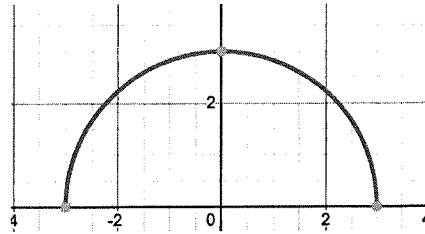
List of functions

- Linear
 - Quadratic
 - Cubic
 - Cube root
 - Square root
 - Rational ($1/x$ and $1/x^2$)
 - All six trig functions
 - Exponential function $\{(-1, 1/b), (0, 1), (1, b)\}$
 - Logarithmic function $\{(1/b, -1), (1, 0), (b, 1)\}$
 - Absolute Value Function
 - All six inverse trig functions (not required to graph, must know domain and range, asymptote for \tan^{-1})
6. If we have several hurricane days, we will have to do a mock exam on a Saturday. Date TBD.

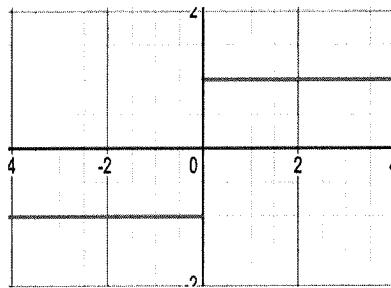
7. AP Calculus Exam TBD

8. In addition to reviewing the functions listed, you also need to memorize:

a. $y = \sqrt{r^2 - x^2}$ is a semi-circle, where r is the radius;



b. $y = \frac{|x|}{x}$,



c. Geometry:

1. Cylinder-is the radius constant?
2. Cone-is the radius Constant?
3. What is the area of a circle?
4. What is the circumference of a circle?
5. Know the area and volume formulas that you should have memorized in Geometry.
6. If a dimension is constant (fixed), does it change over time?

Directions: Beginning in cell #1, read the question and show the work necessary to answer it (attach separate sheets if necessary). Search for your answer and call that cell #2. Continue in this manner until you complete the circuit. Note: The last question will not have a match!

<p># 1 Find the slope of the line which connects the point $(b, 3b)$ to the point $(3b, 6b)$. [Note: $b \neq 0$.]</p>	<p>Answer: $\frac{-1+\ln 3}{2}$ # _____ The graph of $y = 2 \sin(3x - \frac{\pi}{2})$ has an amplitude of _____, a period of _____, and a phase shift of _____ to the _____ (left/right) when compared to the graph of $y = \sin x$.</p>
<p>Answer: $\frac{2e}{1-e}$ # _____ As x grows infinitely large, the value of $h(x) = \frac{2x}{5x+8}$ approaches what number?</p>	<p>Answer: $4/5$ # _____ Find the average rate of change of $w(x) = 3x^2 + 1$ over the interval $[-1, 4]$.</p>
<p>Answer: 75 # _____ For $\frac{\pi}{2} \leq A \leq \pi$, $\sin A = \frac{3}{5}$. Find $\sin(2A)$.</p>	<p>Answer: 9 # _____ If $f(x) = \ln x$ and $g(x) = e^{x+1}$, find $f(g(2)) - g(f(e))$.</p>
<p>Answer: 21 # _____ $f(x) = g^{-1}(x)$ and $g(x) = \frac{2x}{x-1}$; $f(5) = ?$</p>	<p>Answer: $(-\infty, 2) \cup (2, \infty)$ # _____ $\log_{10} 25 + \log_{10} 4 =$</p>
<p>Answer: $[-2, 2]$ # _____ Solve for x: $e^{2x+1} - 3 = 0$</p>	<p>Answer: $x = -3$ # _____ State the domain of $y = \ln(x - 2)$.</p>
<p>Answer: $2/5$ # _____ The expression $3x^2$ is used to calculate the slope at any point on the graph of the function $g(x) = x^3 - 1$. Write the equation of the line tangent to $g(x)$ at its x-intercept.</p>	<p>Answer: $3/2$ # _____ The linear function $f(x)$ is parallel to the line $y = \frac{4}{5}x - 7$ and passes through the point $(-5, 0)$. What is $f(-6)$?</p>

<p>Answer: $-4/5$ # _____ The quadratic function $g(x)$ has a vertex at $(-5, 0)$ and y-intercept of $(0, -5)$. What is $g(1)$?</p>	<p>Answer: 2 # _____ The graph of $g(x) = -\sqrt{4-x^2}$ is a semicircle in quadrants III and IV. Find the domain of $g(x)$.</p>
<p>Answer: 4 # _____ Simplify the expression $\frac{x^3+125}{x+5}$ and then evaluate the resulting expression for $x = -5$.</p>	<p>Answer: 26 # _____ Find $x^2 - y^2$ given that $x + y = 7$ and $x - y = 3$.</p>
<p>Answer: $3 - e^2$ # _____ Given $f(x) = x^2 + 5$, find $\frac{f(3+h)-f(3)}{h}$ ($h \neq 0$).</p>	<p>Answer: 36 # _____ State the range of $w(x) = \frac{2x+1}{x+3}$.</p>
<p>Answer: $x > 2$ # _____ $81^{\frac{3}{4}} + 8^{\frac{2}{3}} + 125^{\frac{1}{3}}$</p>	<p>Answer: $-24/25$ # _____ The graphs of $g(x) = \ln(x + 3)$ and $f(x) = \frac{2x+1}{x+3}$ have the same vertical asymptote. What is it?</p>
<p>Answer: $5/3$ # _____ Solve for x: $\ln(x) - \ln(x + 2) = 1$</p>	<p>Answer: $y = 3x - 3$ # _____ Evaluate $g(x) = 5\sin x + \cos(2x)$ for $x = \frac{\pi}{2}$.</p>
<p>Answer: $-36/5$ # _____ Find the average rate of change of the function $p(x) = \frac{4}{5}x - 2$ from $x=0$ to $x=15$.</p>	<p>Answer: $6 + h$ # _____ If the perimeter of a rectangle is 68 and the width is 10, find the length of a diagonal.</p>

TI-83/84 Plus Graphing Calculator Worksheet

The graphing calculator is set in the following WINDOW, MODE, and Y=, settings. Resetting your calculator brings it back to these original settings.

<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">WINDOW</div> <div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> WINDOW Xmin=-10 Xmax=10 Xscl=1 Ymin=-10 Ymax=10 Yscl=1 Xres=1 </div>	<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">MODE</div> <div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE FUDGE PAR POL SEQ CONNECTED DOT SEQUENTIAL SIMUL REAL a+bi P e^iθ FULL HORIZ G-T SET CLOCK 10:22:02 1:07 AM </div>	<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">Y=</div> <div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> Plot1 Plot2 Plot3 Y1= Y2= Y3= Y4= Y5= Y6= Y7= </div>	<p>Note that all Plots are NOT highlighted. If any of them is highlighted, then use the arrow keys to go up / and right</p> <div style="text-align: center;"> </div> <p>Press to deselect ENTER</p>
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WINDOW Notation x : [x_{min} , x_{max} , x_{scl}] and y : [y_{min} , y_{max} , y_{scl}]
 Original Setting x : [-10, 10, 1] and y : [-10, 10, 1]

Resetting Calculator to Factory Setting:

- when the user have used the calculator in various ways and it is difficult to go back to the original setting.
- when the user lend the calculator to others and they have messed up the original setting.
- this should be done before a test or after you lend the calculator to a friend

^{2nd} MEM +

1: About
 2: Mem Mgmt/Del...
 3: Clear Entries
 4: ClrAllLists
 5: Archive
 6: UnArchive
 7: Reset...

Select Option 7
ENTER

Select Option 1
ENTER

1: All RAM...
 2: Defaults...

This will also delete all your entries like equations in Y= screen as well as data in the STATS screen

Adjusting WINDOW of a graph:

Sometimes, a graph needs to be set with a customize WINDOW. This is similar to setting the intervals and the ranges for both x - and y - axis.

Example 1: Graph $y = -2x^2 + 5x + 15$.

<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">Y=</div> <div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> Plot1 Plot2 Plot3 Y1=-2X^2+5X+15 Y2= Y3= Y4= Y5= Y6= Y7= </div>	<p>To enter negative sign, press</p> <div style="border: 1px solid black; padding: 5px; width: 30px; margin: 0 auto;">(-)</div> <p>To enter X, press</p> <div style="border: 1px solid black; padding: 5px; width: 30px; margin: 0 auto;">X,T,θ,n</div>	<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">GRAPH</div>	<div style="border: 1px solid black; padding: 5px; background-color: #f0f0f0; margin-bottom: 5px;">ZOOM</div> <div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> 0: MEMORY 1: ZDecimal 2: ZSquare 3: ZStandard 4: ZTrig 5: ZInteger 6: ZoomStat 7: ZOOMFit </div>	<p>Scroll down with and press ENTER or Select Option 0</p>
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Note: We use the subtraction button between terms. Otherwise, we use for negative signs.

	<div style="border: 1px solid black; padding: 5px; font-family: monospace; font-size: 0.8em;"> WINDOW Xmin=-10 Xmax=10 Xscl=1 Ymin=-235 Ymax=18.123585... Yscl=1 Xres=1 </div>	<p>The ZoomFit option does not give a neat WINDOW setting, but it allows us to see the whole graph</p>
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To quickly reset the original WINDOW setting without resetting the entire calculator:

ZOOM

Scroll down with and press **ENTER** or Select Option 6

WINDOW

Note the WINDOW goes back to the original setting.

Now, we try using a customize WINDOW setting to x: [-10, 10, 1] and y: [-20, 20, 1].

WINDOW

GRAPH

Note that now the graph fits nicely.

Example 2: Using the graph $y = -2x^2 + 5x + 15$ from the previous example,

- Create a table of values starting at $x = -3$ with an increasing interval of 0.5.
- Trace the graph and find the value of y when $x = 5$ from the graph.
- What is the y -intercept of this graph?
- Determine the x -intercepts.
- Give the coordinates of where the maximum value of this graph occurs.
- Solve $-2x^2 + 5x + 15 > 0$ and then solve $-2x^2 + 5x + 15 \leq 0$.

a. To create and customize a Table of Values:

TBLSET WINDOW

Set Table Start to -3

Set Table Interval to 0.5

We may scroll up and down using

TABLE GRAPH

X	Y1
-3	-18
-2.5	-10
-2	-3
-1.5	3
-1	12
-0.5	15
0	15

X = -3

b. To Trace along a Graph and find a Y-value from an X-value:

GRAPH

TRACE

The equation is displayed on top.

Note the blinking cursor and the value of the current x and y.

Enter 5 to input x-value **ENTER**

Y-value of -10 is shown

c. To find y -intercept, let $x = 0$

Note the y -intercept of a quadratic equation is its constant value after we manipulate it to $ax^2 + bx + c = 0$.

TRACE

Enter 0 to input x -value **ENTER**

y -value of -15 is shown

d. To find x -intercept, let $y = 0$: This means using the ZERO function.

2nd **CALC** **TRACE**

Select Option 2

Use **◀** and take the cursor to the left of the first x -intercept. **ENTER**

Left Bound? $N = -2.340426$ $Y = -7.657311$

Press **ENTER** again.

Right Bound? $N = -1.276596$ $Y = 5.3576279$

Zero $N = -1.760399$ $Y = 0$

Zero = x -intercept = Solution = Root

Use **▶** and take the cursor to the right of the first x -intercept. **ENTER**

Zero $N = 4.2603986$ $Y = 0$

Do the same steps for the second x -intercept.

Note the two little triangles that appear. They indicate the calculator will find the x -intercept within that range.

Because the original quadratic equation, $y = -2x^2 + 5x + 15$, is not factorable, these solutions are the decimal equivalents of the roots found from the quadratic formula. However, we prefer the exact values from the quadratic formula to their decimal equivalents.

e. To find the coordinates of the Maximum (or the Minimum) of a Graph:

2nd **CALC** **TRACE**

Select Option 3 for Minimum
Select Option 4 for Maximum

Use **◀** and take the cursor to the left of the Maximum point. **ENTER**

Left Bound? $N = -.4255319$ $Y = 12.510186$

Press **ENTER** again.

Right Bound? $N = 2.7659575$ $Y = 13.528746$

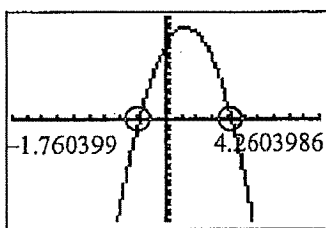
Maximum $N = 1.2499996$ $Y = 18.125$

Use **▶** and take the cursor to the right of the Maximum point. **ENTER**

f. **Solve Inequalities from Graphing:** $(-2x^2 + 5x + 15 > 0)$ and $(-2x^2 + 5x + 15 \leq 0)$

GRAPH

$x: [-10, 10, 1]$
and
 $y: [-20, 20, 1]$



when $y > 0$
(positive y -values)

when $y = 0$
(all y -values of x -axis = 0)

when $y < 0$
(negative y -values)

$$x\text{-intercepts} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(-2)(15)}}{2(-2)} = \frac{-5 \pm \sqrt{145}}{-4} = \frac{5 \pm \sqrt{145}}{4}$$

$$x = \frac{5 - \sqrt{145}}{4} \approx -1.760399 \qquad x = \frac{5 + \sqrt{145}}{4} \approx 4.2603986$$

For $-2x^2 + 5x + 15 > 0$, it is the same as when $y > 0$.

Approx Solution: $-1.760399 < x < 4.2603986$

Exact Solution: $\frac{5 - \sqrt{145}}{4} < x < \frac{5 + \sqrt{145}}{4}$

For $-2x^2 + 5x + 15 \leq 0$, it is the same as when $y \leq 0$. Approx Solution: $x \leq -1.760399$ or $x \geq 4.2603986$

Exact Solution: $x \leq \frac{5 - \sqrt{145}}{4}$ or $x \geq \frac{5 + \sqrt{145}}{4}$

Example 3: Solve $-2x^2 + 5x = -15$ using the INTERSECT function.

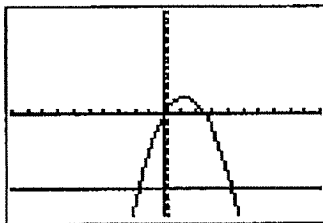
Using the INTERSECT function:

Y= Enter the two sides of the equation as Y_1 and Y_2

Plot1 Plot2 Plot3
 $Y_1 = -2X^2 + 5X$
 $Y_2 = -15$
 $Y_3 =$
 $Y_4 =$
 $Y_5 =$
 $Y_6 =$
 $Y_7 =$

GRAPH

$x: [-10, 10, 1]$
and
 $y: [-20, 20, 1]$

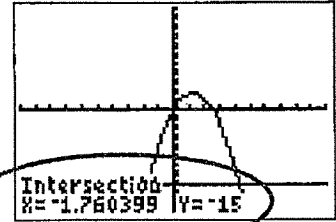
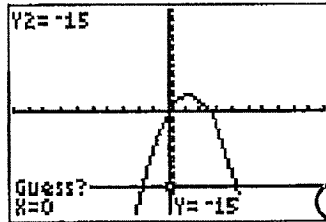
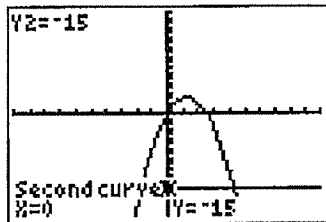
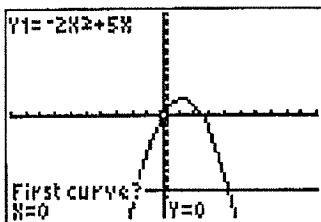


2nd

CALC
TRACE

Select Option 5

- 1: value
- 2: zero
- 3: minimum
- 4: maximum
- 5: intersect
- 6: dy/dx
- 7: ∫f(x)dx

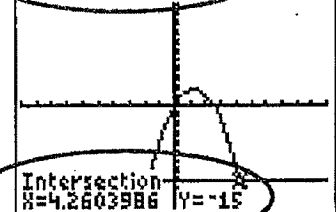


Take cursor close to the first intersecting point

ENTER **ENTER** **ENTER**

Note that solutions for the equation, $-2x^2 + 5x = -15$, are the same as the zeros for $y = -2x^2 + 5x + 15$.

Do the same steps for the second intersecting point.



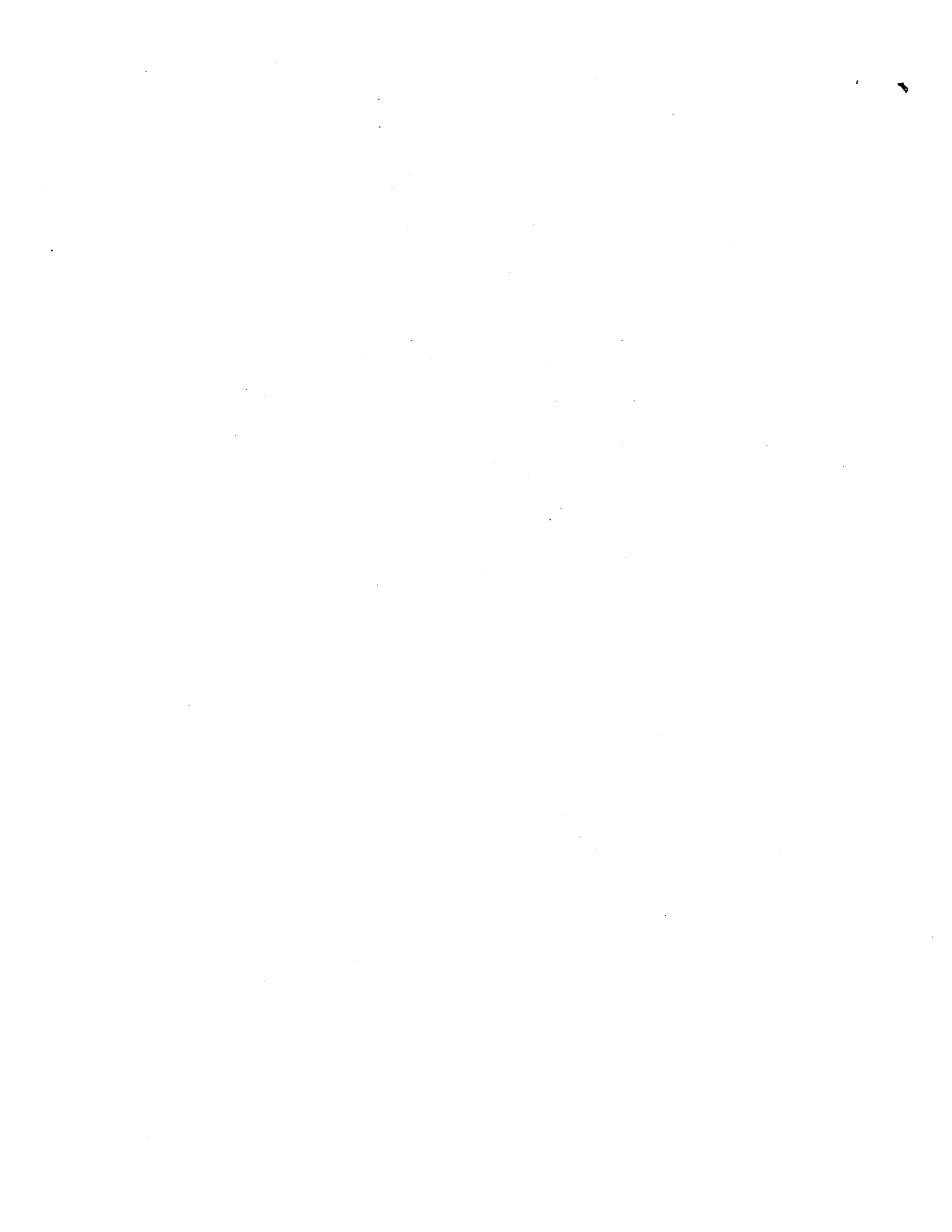
Exercise Questions

1. Graph $y = x^2 + 6x - 16$. Adjust the WINDOW to properly fit the graph.
 - a. Trace the graph and find the value of y when $x = -7$ from the graph.
 - b. What is the y -intercept of this graph? How is the answer compared to the constant of the equation?
 - c. Determine the x -intercepts. How are they compared to solving the equation by factoring?
 - d. Give the coordinates of where the minimum value of this graph occurs.
 - e. Solve $x^2 + 6x - 16 \geq 0$.
 - f. Solve $x^2 + 6x - 16 < 0$.

2. Solve all real solutions $x^3 + 3x^2 - 7x = 15$ to two decimal place by graphing $y = x^3 + 3x^2 - 7x - 15$ and determine its zeros. Adjust WINDOW accordingly.
 - a. Why is find the zeros of $y = x^3 + 3x^2 - 7x - 15$ the same as solving the equation $x^3 + 3x^2 - 7x = 15$?
 - b. Solve the equation, $x^3 + 3x^2 - 7x = 15$, again by using the intersect function of the calculator.
 - c. Give the coordinates (to the two decimal place) where the minimum value of this graph occurs.
 - d. Solve $x^3 + 3x^2 - 7x - 15 < 0$.

3. A number people were shipwrecked on an island. The population of the island slowly grew for 20 years until a passing boat rescued the people. The population on the island can be modeled by the formula, $P = 200(1.1)^t$, where P is the number of years on the island and t is the years that they have been shipwrecked.
 - a. Why is $0 \leq x \leq 20$ an appropriate x range for your window?
 - b. What is an appropriate y range? How will ZOOMF it set a good range for you after you have put in the x range (we used this on the last worksheet)?
 - c. How many people were originally shipwrecked? What time is this?
 - d. What is the population after 5 years? 18 years?
 - e. When is the population 300? When is it 1000?

Answers



Beginning in cell #1, use a combination of analytic methods and a graphing calculator to solve the problem. Show how you arrived at your answer, even if a lot of your work was done on the calculator. Hunt for your answer and call this problem #2. Continue in this manner until you complete the circuit. Note: Answers are rounded or truncated to three decimal places. Also, make sure you know HOW to do these on the test when there are no answer choices!

<p>Answer: 0.510 #1 Find the average rate of change for the function $f(x) = 3e^{-x}$ from $x = -1$ to $x = 7$.</p>	<p>Answer: 1.771 # _____ The function $r(x) = \frac{x+2}{2x-3}$ has a horizontal asymptote of $y =$ _____.</p>
<p>Answer: -1.750 # _____ Find $f(g(-\frac{4\pi}{7}))$ if $f(x) = \begin{cases} x - 2, & x \leq 0 \\ \frac{3}{x}, & x > 0 \end{cases}$ and $g(x) = \tan x$.</p>	<p>Answer: 5.832 # _____ Find the zero of $f(x) = 3 - 2^x$.</p>
<p>Answer: 1.585 # _____ Suppose the number of cases of a rare disease is able to be reduced by 25% annually. If there are 4000 cases nationwide, how many years will it take to reduce the number of cases to 300?</p>	<p>Answer: 1.500 # _____ The graph of an exponential function, $y = a \cdot b^x$, passes through the points (1, 1) and (2, 3.5). Find the value of a.</p>
<p>Answer: 0.500 # _____ If $f(g(x)) = g(f(x)) = x$, and $g(x) = 2 + \ln(x + 1)$, find $f(4)$.</p>	<p>Answer: 9.899 # _____ A cone has a height which is one-sixth the radius. If the radius is two, what is the volume of the cone?</p>
<p>Answer: 1.396 # _____ $g(x) = \ln(x - 4)$ and $f(x) = \frac{1}{2}x^2 + 3$. Find $f(g(6))$.</p>	<p>Answer: 0.685 # _____ A drug is administered intravenously for eight hours, $0 \leq t \leq 8$, and the function $f(t) = 32 - 8.2\ln(1 + 2t)$ gives the number of units of the drug in the body after t hours. How many units are present after 7 hours (at time $t = 7$)?</p>

<p>Answer: 9.004 # _____ What is the period of $y = \sin(4x)$?</p>	<p>Answer: -1.019 # _____ For $g(x) = -3x^2 + 5.2x + 7$, find the maximum value of the function.</p>
<p>Answer: 1.760 # _____ Solve for θ, $\frac{3\pi}{2} \leq \theta \leq 2\pi$. $\cos\theta = 0.9$</p>	<p>Answer: 0.456 # _____ What is the minimum value of $y = -3\cos t + 1.25$?</p>
<p>Answer: 9.794 # _____ The function $v(t) = -9.8t + 5$ gives the instantaneous velocity (In m/sec) of an object thrown upward with an initial velocity of 5 m/sec. At what time t does the object start falling?</p>	<p>Answer: 3.240 # _____ Solve the non-linear system $\begin{cases} y = \sqrt{x+2} \\ y = 1.1x^5 \end{cases}$. To advance in the circuit, locate the y-coordinate of the solution.</p>
<p>Answer: 9.253 # _____ An isosceles right triangle has a leg of 7 cm. What is the length of the hypotenuse, in cm?</p>	<p>Answer: 6.389 # _____ Solve $\sec(3x) = 5$ on the open interval $(0, \frac{\pi}{6})$.</p>
<p>Answer: 0.286 # _____ $\log_3 7 = ?$</p>	<p>Answer: 1.571 # _____ The function $f(x) = \frac{x+2}{2x-3}$ has a vertical asymptote at $x = \underline{\hspace{2cm}}$.</p>