

Things to Know for Calculus

TRIGONOMETRY

Trig Functions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

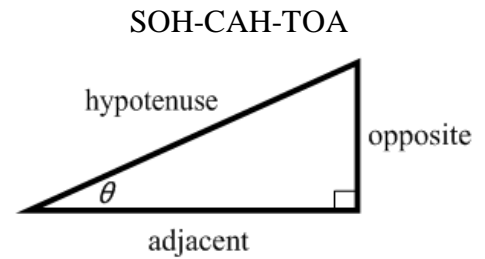
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Reciprocal Functions

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

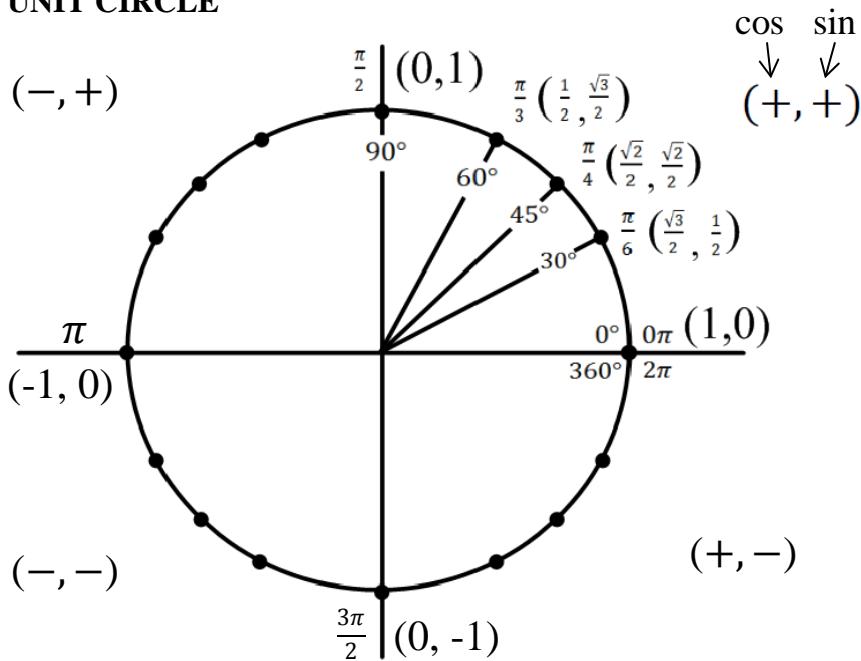
$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$



TEST ONLY USES RADIANS!

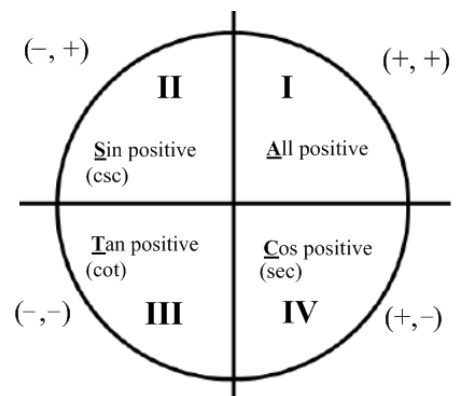
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

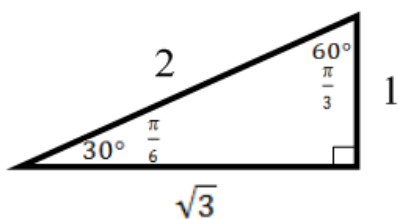
All Students Take Calculus



SPECIAL RIGHT TRIANGLES

30° – 60° – 90° Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

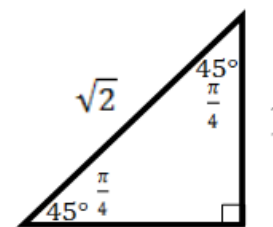


Find $\tan\left(\frac{\pi}{6}\right)$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \text{ simplify to } \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$$

45° – 45° – 90° Triangles

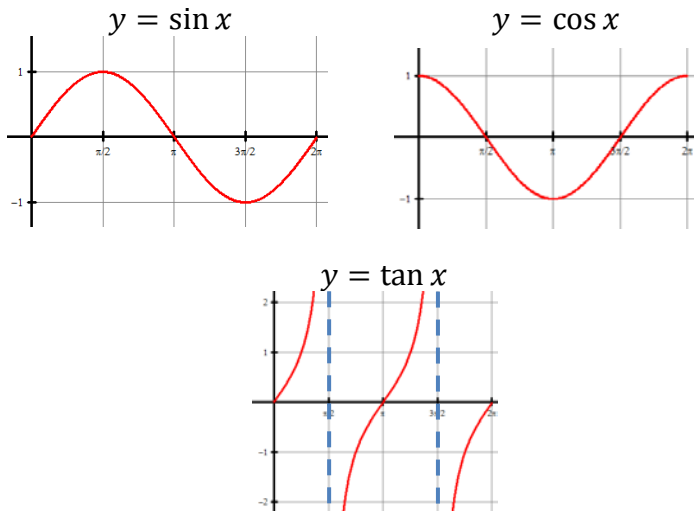
Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles



Find $\sin\left(\frac{\pi}{4}\right)$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \text{ simplify to } \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$

Graphs of trig functions



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin \theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$
that means $\theta = \frac{\pi}{3} \pm 2\pi n$ or $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function.
Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So... $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. **$\sin^2 x + \cos^2 x = 1$**

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

$$\text{Triangle} = \frac{1}{2}bh$$

$$\text{Circle} = \pi r^2$$

$$\text{Trapezoid} = \frac{1}{2}(b_1 + b_2)h$$

SURFACE AREA

$$\text{Sphere} = 4\pi r^2$$

LATERAL AREA

$$\text{Cylinder} = 2\pi rh$$

VOLUME

$$\text{Sphere} = \frac{4}{3}\pi r^3$$

$$\text{Cylinder} = \pi r^2 h$$

$$\text{Cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Prism} = Bh$$

$$\text{Pyramid} = \frac{1}{3}Bh$$

B is the area of the base

CIRCUMFERENCE

$$\text{Circle} = 2\pi r$$

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

Parallel Lines

Have the same slope

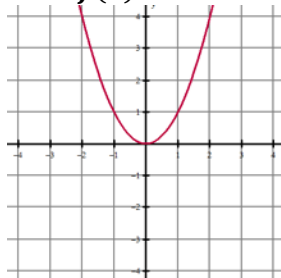
Perpendicular Lines

Have the opposite reciprocal slopes

Functions

Quadratic Function

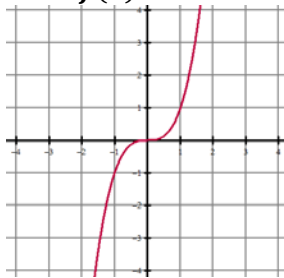
$$f(x) = x^2$$



$$y = a(x - h)^2 + k$$

Cubic Function

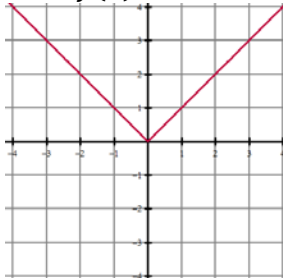
$$f(x) = x^3$$



$$y = a(x - h)^3 + k$$

Absolute Value

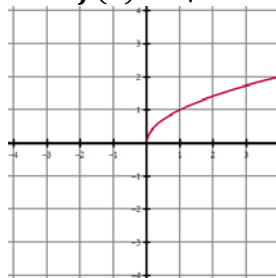
$$f(x) = |x|$$



$$y = a|x - h| + k$$

Square Root Function

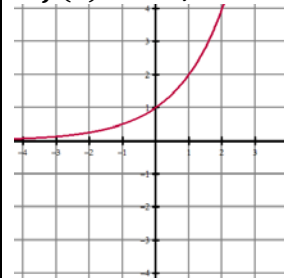
$$f(x) = \sqrt{x}$$



$$y = a\sqrt{x - h} + k$$

Exponential Function

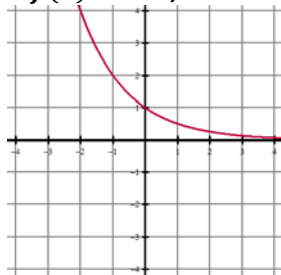
$$f(x) = b^x, b > 1$$



$$y = a \cdot b^{(x-h)} + k$$

Exponential Function

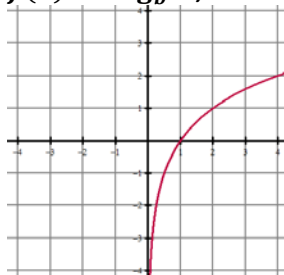
$$f(x) = b^x, b < 1$$



$$y = a \cdot b^{(x-h)} + k$$

Logarithmic Function

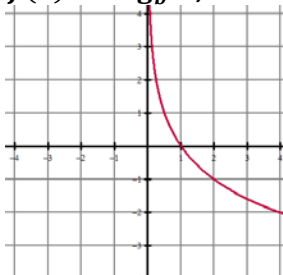
$$f(x) = \log_b x, b > 1$$



$$y = a \log_b(x - h) + k$$

Logarithmic Function

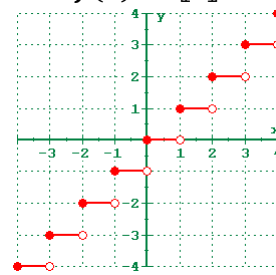
$$f(x) = \log_b x, b < 1$$



$$y = a \log_b(x - h) + k$$

Greatest Integer

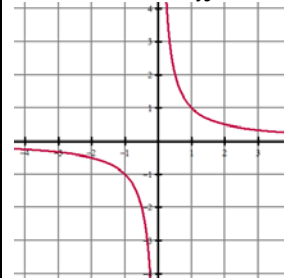
$$f(x) = \llbracket x \rrbracket$$



$$y = a \llbracket x - h \rrbracket + k$$

Rational Function

$$f(x) = \frac{1}{x}$$



$$y = \frac{a}{x - h} + k$$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4

$$y = x^2 + 4$$

Move down 3

$$y = x^2 - 3$$

Move left 2

$$y = (x + 2)^2$$

Move right 1

$$y = (x - 1)^2$$

Move left 2 and down 3

$$y = (x + 2)^2 - 3$$

To flip (reflect) the function vertically $y = -x^2$

To flip (reflect) the function horizontally $y = (-x)^2$

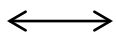
So $f(x) = -\sqrt{x - 3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis () versus closed []

Inequality

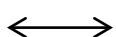
$$-3 < x \leq 5$$



Interval

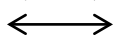
$$(-3, 5]$$

$$-3 \leq x \leq 5$$



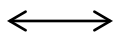
$$[-3, 5]$$

$$-3 < x < 5$$



$$(-3, 5)$$

$$-3 \leq x < 5$$

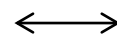


$$[-3, 5)$$

Infinity is always open parenthesis

Inequality

$$x < 3$$



Interval

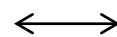
$$(-\infty, 3)$$

$$x \leq 3 \text{ or } x > 5$$



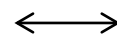
$$(-\infty, 3] \cup (5, \infty)$$

$$x \neq 3$$



$$(-\infty, 3) \cup (3, \infty)$$

$$\text{all Real numbers}$$



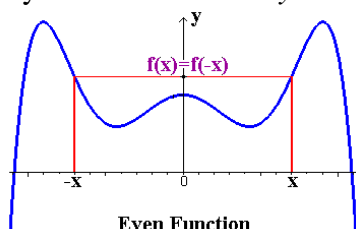
$$(-\infty, \infty)$$

Even and Odd Functions

EVEN

$$f(-x) = f(x)$$

Symmetric about the y-axis

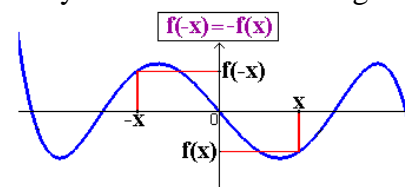


Even Function

ODD

$$f(-x) = -f(x)$$

Symmetric about the origin



Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically

You can't divide by zero

You can't square root a negative

$$y = \sqrt{2x + 5}$$

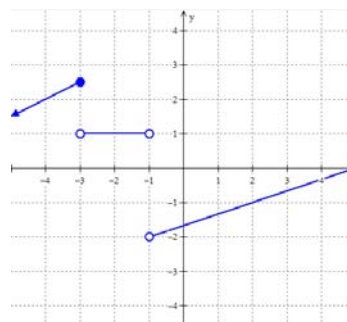
$$D: \left[-\frac{5}{2}, \infty\right)$$

$$y = \frac{x^2 - 1}{x^2 + 7x + 12}$$

$$D: (-\infty, -4)(-4, -3)(-3, \infty)$$

Graphically

Just look at it



$$D: (-\infty, -1)(-1, 5]$$

$$R: (-\infty, 2.5]$$

Finding zeros

Must be able to factor and use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

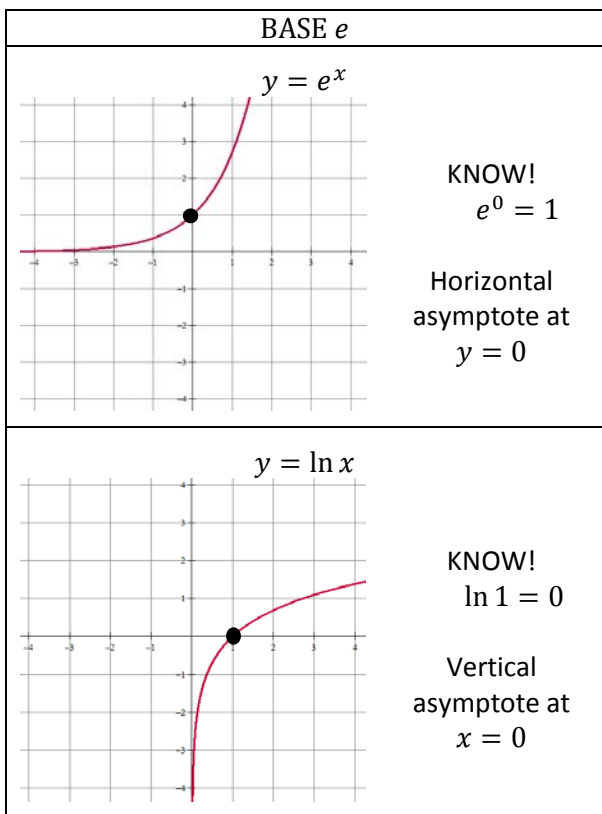
Special products

Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logarithmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x " or "log base b of x ". So...



$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln x$
	Common Log	$\log_{10} x = \log x$